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A Two-Unit Cold Standby System with Rest of Repairman and Correlated Failures and Repairs

Abstract

This paper analyses a two unit cold standby system with rest of repairman. The failure and repair times of each unit are assumed to be correlated by taking their joint density as bivariate exponential. Using the regenerative point technique various reliability characteristics of interest are obtained. The behavior of MTSF have also been studied through graphs.

Keywords: Regenerative Point Technique, Standby System, MTSF and Availability Analysis.

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Introduction

Various researchers including references [1-4] have analysed two unit cold standby systems under the assumption that the repairman does not need rest during the repair of a failed unit. This assumption may not hold in many real life systems. In fact, repairmen may feel need of rest whenever he is busy to repair a failed unit. In these systems [1-2] it has been assumed that the failure and repair time distribution are independent. But, in practical life, it is also observed that there are some systems [3-4] with correlated failure and repair time. Keeping these facts in view, we analyse here a two-unit cold standby system with rest of repairmen and correlated failure and repair time.

Aim of the Study

The object of present paper to obtain various measures of the system effectiveness such as mean time to system failure , steady state availability and expected busy period of the repairman using regenerative point technique. Also find out the impact of correlation on the system performance.

Model Description

The system consists of two identical units. Initially one unit is operative and the other is cold standby. Repairman does not need rest when both the units are in good state. Upon failure of the operative unit, the standby unit becomes operative instantaneously and failed unit undergoes for repair. After repair, a unit works like a new one. Repairmen may feel need of rest during the repair of a failed unit. It is assumed that a repairman needs rest with probability p while repairing a failed unit. The rest time distribution is negative exponential with parameter α. The joint distribution of failure (x) and repair (y) times of each unit is bivariate exponential (λ , μ).

Nation and States

No/Ns : unit is normal mode and operative/standby

Fr/Frc/Fw : failed unit is under repair/repair is continued from earlier state/ waiting for repair

FR/FRc : repairmen feels need for rest during the repair of the failed unit/ feel of need for rest is continued from previous state

Now, using the above symbols, we have following states of system:

Up states $\underline{S}_0 = (N_0,N_s)$, $\underline{S}_1 = (F_r, N_0)$,
$\underline{S}_3 = (FR , N_0)$

Down states $S_3 = (F_{rc}, F_w)$, $S_4 = (FR_c,F_w)$
$\underline{S}_5 = (F_w, F_w)$, $\underline{S}_6 = (F_r,F_w)$

The underlined states are regenerative. Transition diagram is shown in figure1 .

Transition Probabilities and Sojourn Times

The state transition probabilities are :

$p_{01} = 1$,$p_{10} = \frac{q_\mu}{\lambda+\mu}$, $p_{12} = \frac{p_\mu}{\lambda+\mu}$, $p_{15}^{(3)} = \frac{p_\lambda}{\lambda+\mu}$,

$p_{11}^{(3)} = \frac{p_\lambda}{\lambda+\mu}$, $p_{21} = \frac{\alpha}{\alpha+\lambda(1-r)}$, $p_{26}^{(4)} = \frac{\lambda(1-r)}{\alpha+\lambda(1-r)}$, $p_{56} = 1$, $p_{61} = q$, $p_{65} = p$

The mean sojourn times in the various states are:

$T_0 = \frac{1}{\lambda(1-r)}$, $T_1 = \frac{1}{(\lambda+\mu)(1-r)}$, $T_2 = \frac{1}{\alpha+\lambda(1-r)}$, $T_3 = T_5 = \frac{1}{\alpha}$, $T_6 = \frac{1}{\mu(1-r)}$

Mean Time to System Failure

Considering the failed states S3 and S4 as absorbing, we have the following equations:

$\Pi_0(t) = Q_{01}(t)\ s\ \Pi_1(t)$

$\Pi_1(t) = Q_{10}(t)\ s\ \Pi_0(t) + Q_{12}(t)\ s\ \Pi_2(t) + Q_{13}(t)$

$\Pi_2(t) = Q_{21}(t)\ s\ \Pi_1(t) + Q_{24}(t)$ (1-3)

Taking the laplace-stieltjes transform of above equation and using the well- known formula for MTSF,we get:

$MTSF = [T_O(1-p_{12}p_{21}) + T_1 + T_2]|(1-p_{10} - p_{12}p_{21})$ (4)

Availability Analysis

From the arguments used in the theory of regenerative process, we get:

$A_0(t) = M_0(t) + q_{01}(t)© A_1(t)$

$A_1(t) = M_1(t) + q_{10}(t)©A_0(t) + q_{12}(t) © A_2(t) + q_{11}^{(3)}(t) © A_1(t) + q_{15}^{(3)}(t)\ (t)\ ©A_5(t)$

$A_2(t) = M_2(t) + q_{21}(t)\ ©A_1(t) + q_{26}^{(4)}(t)\ (t)\ ©A_6(t)$

$A_5(t) = q_{56}(t)\ ©A_6(t)$

$A_6(t) = q_{11}(t)©A_1(t) + q_{65}(t)© A_5(t)$ (5-9)

Where,

$M_0(t) = \exp[-\lambda(1-r)t]$

$M_1(t) = \exp[-(\lambda+\mu)(1-r)t]$

$M_2(t) = \exp[-\{\alpha+\lambda(1-r)\}t]$

By taking laplace transform of above equations, we get $A_0^*(s)$. Using this result, the steady state availability of the system is:

$A_0 = \lim_{t\to\infty} A_O(t) = \lim_{s\to 0} s\, A_0^*(s) = N/D$ (10)

Where,

$N = (1-p)[p_{10}T_0 + p_{01}(T_1 + p_{12}T_2)]$ and

$D = (1-p)[p_{10}T_0 + T_1 + p_{13}T_3 + p_{12}(T_2 + p_{24}T_4)] + (p\ p_{12}p_{26}^{(4)}(t) + p_{15}^{(3)}(t))T_5 + (q_{15}^{(3)}(t) + p_{12}p_{26}^{(4)}(t))T_6$

Busy Period Analysis

By sample probabilistic reasoning, we have

$B_0(t) = q_{01}(t) © B_1(t)$

$B_1(t) = W_1(t) + q_{10}(t) © B_0(t) + q_{12}(t) © B_2(t) + q_{11}^{(3)}(t)\ (t) © B_1(t) + q_{15}^{(3)}(t)\ (t) © B_5(t)$

$B_2(t) = q_{21}(t) © B_1(t) + q_{26}^{(4)}(t)\ (t) © B_6(t)$

$B_5(t) = q_{56}(t) © B_6(t)$

$B_6(t) = W_6(t) + q_{61}(t)\ ©B_1(t) + q_{65}(t)©B_5(t)$ (11-15)

Where,

$W_1(t) = \exp\ [-(\lambda+\mu)(1-r_1)] + q_{11}^{(3)}(t)©[-(\lambda+\mu)(1-r_1)] + q_{15}^{(3)}(t) © \exp(-\alpha t)$

$W_6(t) = \exp[-\mu(1-r)t] + q_{61}(t) © \exp[-(\lambda+\mu)(1-r)$

By taking the Laplace transform of the above equations we obtain $B_0^*(s)$, and the steady state probability that the repairman is busy is given by;

$B_0 = \lim_{t\to\infty} B_O(t) = \lim_{s\to 0} s\, B_0^*(s) = N_1|D$

(16)

Where

$N_1 = (1-p)(T_1 + p_{11}^{(3)}T_1 + p_{15}^{(3)}T_5) + (p_{12}p_{26}^{(4)} + p_{15}^{(3)})[(1-p)T_1 + T_6]$

Conclusion

Graphical Interpretation

Trand of MTSF is observed through graphs in Fig.2 w.r.t repair rate (μ) for different values of correlation coefficient(r). It can be observed from the graph that the values of MTSF become higher for higher values of correlation coefficient(r) and repair rate(μ).

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Figure 1
Transition Diagram

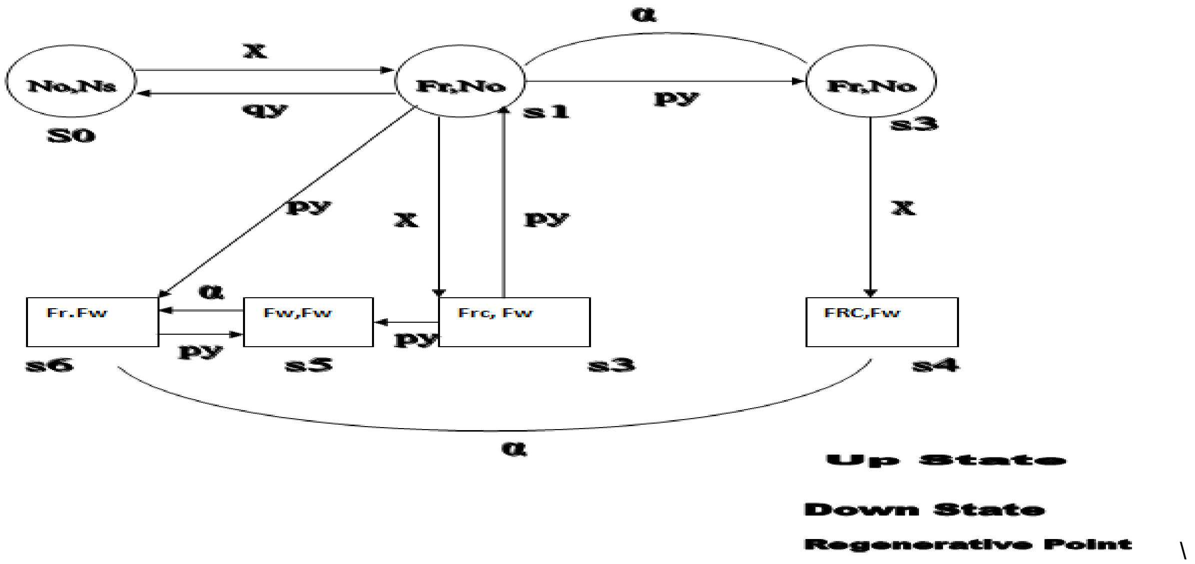

Figure 2
Behavonr of MTSF Wrt to μfor different Values fo r